


UE 2 : "OPTIONAL COURSES"

 ECTS
9 crédits

 Composante
UFR de
mathématiques
et
informatique
(UFR27)

Liste des enseignements

Obligatoire Choix À choix 1 Matière 48.0 Matière 2.0 À choix 1 Choix À choix 1 Choix Obligatoire Matière 54.0 Travaux Dirigés 18.0 Cours Magistral 36.0 Choix À choix 2 Matière 3.5 Matière 42.0 Matière 42.0 Cours Magistral 18.0 Travaux Dirigés 24.0 Matière 42.0 Objectifs : Option pricing in discrete and continuous time, with martingales use and first steps of stochastic calculus. Contenu du cours : Chapter I. Preliminaries 1. Derivative products, description and use: Forward/Future contracts, Options 2. Rates and discounting 3. Arbitrage methods Chapter II. Forward contracts pricing (reminder, in tutorial) Chapter III. Mathematical tools 1. Conditional expectation, martingale. Chapter IV. Option pricing in discrete time 1. N periods binomial model (Cox-Ross-Rubinstein); self-financing strategies, 2. risk-neutral probability, martingale property of the discounted price process, 3. option pricing, delta hedging. Chapter V. Option pricing in continuous time: Black-Scholes model 1. Brownian motion and Ito processes. 2. Quadratic variation of the Brownian motion, 3. Ito integral for a simple process, 4. Extension to the computation of $\int_t^T B_t dB_t$, 5. Ito lemma (heuristic proof). 6. Black-Scholes model 7. Partial differential equation approach, hedging from that equation. 8. Probabilistic approach for European options, 9. Girsanov theorem (particular case), 10. Black Scholes formula, delta computation, use. Références : Hull, Options, futures, and other derivative securities, Prentice-Hall (2018: 10th ed). Baxter, M. and Rennie, A., Cambridge University Press, 1996. Kwok, Y.K., Mathematical models of financial derivatives, Springer, 2nd edition, 2008 (3 first chapters). Jacod, J., Protter, P. (2000) Probability Essentials. Springer. Matière 42.0 Matière 42.0 Objectifs: Martingales and Markov chains in discrete time Prerequisite: Probability with measure: σ -fields, measure space, measurable maps. Non-negative measures, integration of real valued functions. Convergence of sequences of real valued maps. Monotone convergence, Fatou lemma, dominated convergence (Lebesgue). L_p spaces. Probability measure. Random variables. Expectations of r.v. Independence of sub- σ -fields, independence of random variables. Contenu du cours: Conditional expectation, definition, properties Discrete time processes, filtration, stopping time, σ -field of events determined prior to a stopping time Discrete time Martingales, stopped martingales, optional sampling theorem, maximal inequalities, convergence of martingales, regular martingales Markov chains with countable states, conditional independence, Markov property, Markov sequences, transition matrix. Markov chains, communication classes, recurrence and transience, positive states, null states, invariant measures, ergodic properties References Jacques Neveu: Bases Mathématiques de la théorie des probabilités Jacques Neveu: Martingales en temps discret Lacroix, P. Priouret, Cours: J. Lacroix, Probabilités approfondies, Université Pierre et Marie Curie, Master de Mathématiques, 2005-2006 Jean Jacod, Chaînes de Markov, Processus de Poisson et Applications, Université Pierre et Marie Curie, DEA de Probabilités et Applications, 2003-2004